Technical Report

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An optional alternate background noise correction sensitive to the steadiness of background noise

Technical Report

Rue du Rhône 114 CH-1204 Geneva T: +41 22 849 6000 F: +41 22 849 6001

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Introduction

This Technical Report presents an alternate background noise correction to the standard background noise correction in ECMA-74 and ISO 7779. The alternate background noise correction may permit the reporting of lower noise emissions for quiet products than those reported using the standard background noise correction. The alternate background noise correction depends on background noise proximity to the measured source level, like the standard background noise correction, but also on the steadiness of background noise. The alternate background noise correction is used in the same manner as the standard background noise corrections in ISO 3741, ISO 3744, ISO 3745, and ISO 11201 and can be applied to A-weighted or un-weighted band or overall sound power or sound pressure levels.

The purpose of a background noise correction is to remove a background noise contribution from a measured source level, which contains source and background noise contributions. The result is a true source estimate called the background noise-corrected source level. The background noise contribution to the measured source level must be estimated, since measured source and background noise levels are measured over distinct and separate time spans. Background noise fluctuates over time because of uncontrollable acoustic affecting the acoustic environment and uncontrollable electronic and electromagnetic sources measurement instrumentation. Since background noise fluctuates over time, a background noise-corrected source level has uncertainty and may overstate or understate the true source level.

Present standards manage the uncertainty by imposing caps on background noise corrections, the idea being to minimize risk of understating true source level. The applicable standards heuristically cap the background noise correction at 0,46 dB or 1,26 dB, depending on background noise proximity to measured source level, frequency bandwidth, and chamber accuracy grade. An undesirable consequence is that background noise-corrected source levels are not consistent, varying by the standard being followed. Even worse, a grade penalty may occur such that background noise-corrected source levels for engineering accuracy Grade 2 chambers are lower than for precision grade 1 chambers, thereby discouraging use of Grade 1 chambers by manufacturers. Another drawback is that the present standard background noise corrections do not statistically bound the true source level in any stated manner.

The alternate background noise correction answers these shortcomings through a statistical formulation that manages uncertainty. The alternate background noise correction produces a background corrected source level that upper bounds the true source level with 95% confidence. The steadiness of the measured background noise affects the alternate background noise correction, the magnitude of the correction tending to increase with background noise steadiness and proximity to measured source level.

The statistical formulation of the alternate background noise correction has several advantages over the standard background noise corrections.

One advantage is the certainty provided by the statistical formulation, which upper bounds true source level with 95% confidence. Statements about validity accompany the standard background noise corrections but no statistical bounds are given. Present standards deem background corrected source levels obtained from capped corrections to be invalid yet reportable; the report shows these source descriptions to upper bound the true source with unknown confidence. Background corrected source levels obtained from uncapped corrections are deemed valid, even though these source descriptions have high 50% risk of understating the true source, as shown in the report. It is expected that manufacturers and customers will appreciate the source descriptions provided by the alternate background noise correction because they upper bound the true source with known confidence.

Another advantage is that, for low level sources relative to the background, the alternate background noise correction provides background corrected source levels lower than those provided by the standard background noise correction, because of the caps in the standard background noise correction. The caps are present when background noise is within 6 dB of measured source level for engineering Grade 2 and within 10 dB of measured source level for precision Grade 1. Using the alternate correction for the precision Grade 1 method with a steady background, background corrected source levels are approximately 2 dB lower when the measured source level is 3 dB above background level because of equal source and ambient contributions; for the engineering Grade 2 method they are about 1 dB lower. Reduced background corrected source levels result because of the 0.46 dB and 1.3 dB caps for precision and engineering grades, respectively. For source contributions below a steady background, the alternate background correction provides background corrected source levels more than 1 to 2 dB lower, depending on the grade, and as much as 10 dB lower than those provided by the standard background correction.

The reduction of background corrected source levels relative to capped standard background noise corrections depends on background noise steadiness and proximity to measured source level, as well as the standard being followed. A statistical margin term in the alternate background noise correction based on a characterization of measured background noise allows the additional reduction. It is expected that manufacturers will find the alternate background noise correction appealing because it provides background corrected source levels that are minimized while also statistically bounding the true source.

Finally, the alternate background noise correction has the appeal of providing a path towards eliminating the inconsistent background noise corrections and the chamber grade penalty in the applicable standards. By being sensitive to both the steadiness and proximity of background noise to measured source level, the alternate background noise correction upper bounds the true source level in a manner that is applicable to background noise conditions that are stationary across measured source and background noise sampling, without recourse to heuristic caps that can vary by standard. By adopting the alternate background noise correction in place of the standard corrections, future standards will prescribe background corrected source levels for various chambers, frequency weightings and bandwidths that are consistent and comparable to one another and free of the chamber grade penalty.

To be compatible with past applications, it is intended that the alternative background noise correction be used only when it is greater than the standard background noise correction.

This Ecma Technical Report was developed by Technical Committee 26 and was adopted by the General Assembly of December 2017.

An optional alternate background noise correction sensitive to the steadiness of background noise

1 Scope

This Technical Report describes an alternate background noise correction to the standard background noise correction in ECMA-74 and ISO 7779. The alternate background procedure may permit the reporting of lower noise emissions for quiet products than those reported using the standard background noise correction. The alternate background correction depends not only on mean background noise proximity to measured source levels, like the standard background noise corrections of ISO 3741, ISO 3744, ISO 3745, and ISO 11201, but also on the steadiness of the background noise. Background noise fluctuates over time because of uncontrollable acoustic affecting the acoustic environment and uncontrollable electronic and electromagnetic sources measurement instrumentation. Like the standard background noise corrections, the alternate background noise correction tends to increase as the background noise level approaches the measured source level, but the alternate background noise correction also increases with the steadiness of the background noise, unlike the standard background noise corrections.

For low level sources relative to the background, the alternate background noise correction provides background corrected source levels lower than those provided by the standard background noise correction, when background noise is within 6 dB of measured source level for engineering Grade 2 methods and within 10 dB of measured source level for precision Grade 1 methods, because of caps in the standard background noise correction. For the precision Grade 1 method with a steady background, background corrected source levels are approximately 2 dB lower when the measured source level is 3 dB above background level because of equal source and ambient contributions; for the engineering Grade 2 method they are about 1 dB lower. For source contributions below a steady background, the alternate background correction provides background corrected source levels more than 1 to 2 dB lower, depending on the Grade, and as much as 10 dB lower than those provided by the standard background correction.

2 References

The following documents, in whole or in part, are normatively referenced in this document and are indispensable for its application. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies. Full citations of the references are available in the Bibliography.

ISO 3741:2010, *Acoustics — Determination of sound power levels and sound energy levels of noise sources using sound pressure — Precision methods for reverberation test rooms*

ISO 3744:2010, *Acoustics — Determination of sound power levels and sound energy levels of noise sources using sound pressure — Engineering methods for an essentially free field over a reflecting plane*

ISO 3745:2012, *Acoustics — Determination of sound power levels and sound energy levels of noise sources using sound pressure — Precision methods for anechoic rooms and hemi-anechoic rooms*

ISO 11201:2010, *Acoustics — Noise emitted by machinery and equipment — Determination of emission sound pressure levels at a work station and at other specified positions in an essentially free field over a reflecting plane with negligible environmental corrections*

3 Terms and definitions

For the purposes of this document, the following terms and definitions apply.

3.1
background noise
average of the square of the sound pressure of background noise over time (mean square sound pressure)

NOTE Background noise is expressed in square pascals (Pa^2).

3.2
background noise sample difference
difference of two samples of background noise

NOTE Background noise sample difference is expressed in square pascals (Pa^2).

3.3
background noise level
level of the background noise referenced to 20 micro-Pascals

NOTE Background noise is expressed in decibels.

3.4
background noise-corrected source level, $\overline{L_p}$
a true source level estimate obtained by removing an estimate of the background noise contribution to the measured source level

NOTE Background noise-corrected source level is expressed in decibels.

3.5
background noise correction, K_1
the amount by which measured source level is reduced to obtain background noise-corrected source level

NOTE Background noise correction is expressed in decibels.

3.6
mean background noise level, $\overline{L_{p(\mathrm{B})}}$
the mean energy average sound pressure level over all microphone positions for the background noise, (dB)

3.7
mean source sound pressure level, $\overline{L'_{p(\mathrm{ST})}}$
the mean energy average sound pressure level over all microphone positions for the noise source under test, (dB)

3.8
mean square
average of the square of sound pressure over time

NOTE Measured source level is expressed in square pascals (Pa^2).

3.9
measured source level
a measured noise level containing source and background noise contributions

NOTE Measured source level is expressed in decibels.

3.10
true source level
the sound level radiated by a source, without background noise contribution and free of measurement error

NOTE True source level is expressed in decibels.

4 Abbreviations

BNCSL background noise-corrected source level

VCS variance Chi-square

5 Background noise-corrected source level

The background noise-corrected source level (BNCSL) $\overline{L_p}$ is an estimate of the true source level and is given by:

$$\overline{L_p} = \overline{L'_{p(\mathrm{ST})}} - K_1 \tag{1}$$

where $\overline{L'_{p(\mathrm{ST})}}$ is a measured source level containing both source and background noise contributions, and K is a background noise correction. The background noise correction K_1 is either the standard background noise correction K_{1S} according to ISO standards, described in Clause 6, or the alternate background noise correction K_{1A} according to this TR under Clause 7. The background noise correction depends on the difference ΔL_p of the measured source level, which contains a background contribution, relative to the background noise level. The difference $\Delta L_p = \overline{L'_{p(\mathrm{ST})}} - \overline{L_{p(\mathrm{B})}}$, in which background noise level $\overline{L_{p(\mathrm{B})}} = 10\lg(p^2_{(B)} / p^2_0)$ includes a ratio of the background noise $p^2_{(B)}$ to the square of the 20 micro-Pa reference pressure p_0. The background noise is the average of the square of background noise sound pressure over time.

NOTE 1 The horizontal bar above symbols denotes a mean (energy average) over microphone positions.

NOTE 2 The background noise correction K_1 is determined according to either Clause 6 according to ISO 3744, ISO 3745, etc. or according to Clause 7 by the alternate method contained in this TR

6 Standard background noise correction according to ISO standards

The standard background noise correction K_{1S} is given by

$$K_{1S} = \begin{cases} -10\lg(1-10^{-\Delta L_p/10}) & \Delta L_p > \Delta_0 \\ K_{1\max} & \Delta L_p \le \Delta_0 \end{cases} \tag{2}$$

The cap $K_{1\max}$ limits the maximum value of the background noise correction to its value at the measured source to background level difference Δ_0 in decibels. Noise source measurements are deemed valid by the applicable standards when $\Delta L_p > \Delta_0$ and the background noise correction is not capped. The values of $K_{1\max}$ and Δ_0 vary by standard depending on frequency bandwidth and grade of accuracy [1,2,3,4]. In some standards Δ_0 = 6 dB and $K_{1\max}$ = 1,26 dB; in other standards Δ_0 = 10 dB and $K_{1\max}$ = 0,46 dB. The different caps may produce background noise-corrected source levels (BNCSL) that are not consistent across the standards. Moreover, the larger cap is prescribed for engineering accuracy Grade 2 chambers and the smaller cap is prescribed for precision Grade 1 chambers, such that a Grade 1 chamber may yield a higher BNCSL than an engineering Grade 2 chamber. The situation amounts to a grade penalty that may discourage use of Grade 1 chambers by manufacturers, since a lower BNCSL may be obtained in a Grade 2 chamber.

In implementing the standard background noise correction, mean values are typically used for the measured source level $\overline{L'_{p(\mathrm{ST})}}$ and the measured background noise level $\overline{L_{p(\mathrm{B})}}$. No rigorous consideration is given to the fluctuation of background noise over time. A simple thought experiment suggests that background noise correction should increase with steadiness of background noise.

NOTE The symbols $K_{1\max}$ and Δ_0 have been introduced in this report for clarity; they are not found in the applicable standards.

7 Alternate background noise correction according to this TR

The alternate background noise correction depends not only on the mean of the background noise but also the steadiness of the background noise. The alternate background noise correction K_{1A} is given by

$$K_{1A} = \min\left[K_{1U}, \max\left(K_{1L}, K_{1ST}\right)\right] \tag{3}$$

in which K_{1U} and K_{1L} are upper and lower limits imposed on statistical background noise correction K_{1ST}. To limit K_{1A} values to those of the standard correction K_{1S}, for example, K_{1U} would be set to $K_{1\max}$, and K_{1L} would be set to zero. Selection of the upper and lower limits must be done carefully because they change the confidence in the upper bound on the true source level provided by statistical background noise correction K_{1ST}. The upper limit K_{1U} increases the confidence beyond that of K_{1ST} and if set too low produces BNCSL lower than yielded by the standard background noise correction K_{1S}. The lower limit K_{1L} decreases confidence in the upper bound and increases risk of understating the true source level. Specification of the upper and lower limits K_{1U} and K_{1L} in Equation (3) is left for future work.

The statistical background noise correction is given by

$$K_{1ST} = -10\lg\left[1 - 10^{-\Delta/10}\left(1 - \frac{u_\alpha}{2M}\right)\right] \tag{4}$$

in which u_α is a percentile of the background noise sample difference distribution and M is a background noise steadiness, as explained in Annex A. A background corrected source level produced by the statistical background noise correction K_S upper bounds the true source level with confidence value α at which the percentile is evaluated. The percentile u_α is given by

$$u_\alpha = \Phi_{VCS}^{-1}(M,\alpha) \approx \sqrt{8M}\,\Phi_N^{-1}(\alpha) \tag{5}$$

in which Φ_{VCS}^{-1} is the inverse cumulative VCS distribution, and the right hand side is an approximation involving the standard inverse cumulative normal distribution Φ_N^{-1} with zero mean and unit standard deviation. The background noise sample difference arises because a background noise-corrected source level (BNCSL) involves a difference between two samples of background noise. One sample is the background noise contribution to the measured source level; the other sample is the background noise removed from the measured source level by the background noise correction. Annex A shows that the background noise sample difference follows the variance Chi-square (VCS) distribution, which may be approximated by the normal distribution.

The statistical background noise correction K_S reduces to the uncapped standard background noise correction K_1 at the 50% confidence value as may be seen by comparing Equations (2) and (4). The percentile $u_\alpha = 0$ at

$\alpha = 50\%$ because of the symmetry of the background noise sample difference distribution. A BNCSL produced by the standard background noise correction K_1, when uncapped, is therefore equally likely to understate or overstate the true source level. A 50% understatement risk can be undesirable especially as background noise level approaches measured source level and is the reason for the heuristic cap $K_{1\max}$ in the standard background noise correction. The cap decreases the background noise correction and increases the BNCSL along with the confidence of upper bounding the true source level, although this effect is not expressed or quantified in the present standards.

By contrast, the statistical background noise correction manages the situation of comparable source and background noise levels through the margin term containing percentile u_α in Equation (4). The background noise correction decreases with background noise unsteadiness and proximity to measured source level (and increases with background noise steadiness and separation from measured source level).

The alternate background noise correction takes variation of the background noise into account through the background noise steadiness M:

$$M = \frac{\hat{\mu}^2_{p^2}}{\hat{\sigma}^2_{p^2}} \tag{6}$$

The background noise steadiness describes the consistency and uniformity of background noise. The background noise steadiness M increases as consistency and uniformity of the background noise increase and decreases as inconsistency and variability of the background noise increase. Here $\hat{\mu}_{p^2}$ and $\hat{\sigma}^2_{p^2}$ are measured estimates of the mean and standard deviation of the background noise obtained by sampling background noise when the source is not operating. It is important to note that the statistics are taken over the background noise—not the background noise level. Each background noise sample is an average over time of the square of the background noise sound pressure. Given a set of measured background noise levels $\overline{L_{p(B)i}}$ and corresponding background noise samples $p_i^2 = p_0^2 10^{\overline{L_{p(B)i}}/10}$, the estimates of the mean and variance are

$$\hat{\mu}_{p^2} = \frac{1}{N}\sum_{i=1}^{N} p_i^2 \tag{7}$$

$$\hat{\sigma}^2_{p^2} = \frac{1}{N-1}\sum_{i=1}^{N} (p_i^2 - \hat{\mu}_{p^2})^2 \tag{8}$$

where $i = 1 \ldots N$ denotes the measurement sample and reference value p_0 is 20 micro-Pa.

Plots of the VCS distribution of background noise sample difference, along with the normal approximation, are shown for various values of background noise steadiness M in Figure A.1. The normal approximation nearly matches the VCS density for $M \geq 10$. Exact VCS and approximate normal percentiles u_α are compared in Figure A.2 for the confidence value $\alpha = 95\%$ selected for the alternate background noise correction. The normal approximation overstates the VCS percentile by less than 1% at $M = 1$, and the overstatement decreases rapidly with increasing M. The accuracy of the normal approximation, Equation (5), is fortunate for implementation of the alternate background noise correction because of the familiarity of the normal distribution.

Implementing the statistical background noise correction requires measuring, or sampling, the background noise. Background sampling may be done before, after or in between noise source measurements. Background samples may be gathered at the same locations used for source measurements. The background noise samples may then be used to determine the measured background noise steadiness M by Equations (6)-(8).

The question arises as to how many background noise samples are needed to implement the statistical background noise correction. At least two background noise samples are needed to calculate the mean and variance of the background noise. The number of required samples is expected to decrease with increasing steadiness of the background noise for statistical reasons. Indeed, statistical background noise corrections K_S obtained from background noise simulated by Monte Carlo methods were observed to converge with three background noise samples [6]. Only two background noise samples were needed for background noise steadiness $M \geq 100$ [6]. The statistical background noise correction should not be used for steadiness $M < 10$ due to the accuracy of the normal approximation to the VCS distribution above background steadiness of 10; see Figure A.1 and NOTE 4.

These ranges of background noise steadiness are overlaid on the plots in Figure 1, which show the effect of background noise outliers on the steadiness of background noise steadiness measured in an anechoic chamber at different times at three locations [7]. The possibility of background noise outlier effects on the statistical background noise correction was first raised in [8]. The removal of outliers collapses the measured steadiness values along a line for measured steadiness values above $M = 250$ in Figure 1(b).

The guidance provided in [6] for background noise sampling required for the statistical background noise correction appears reasonable in light of the measured background noise data of [7]:

- The statistical background noise correction should not be used for low background steadiness, $M < 10$
- For high background noise steadiness $M \geq 100$, measure at least two background noise samples
- For intermediate background noise steadiness $10 < M < 100$, measure at least three background noise samples

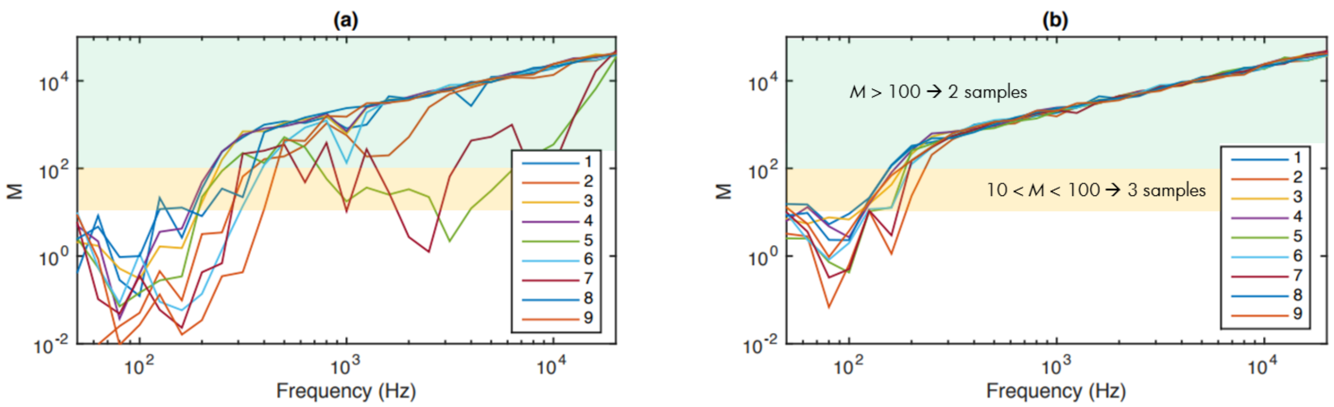


Figure 1 – Background noise steadiness M by location and time: a) all samples and b) outliers removed [7]

NOTE 1 In calculating the background noise steadiness M, the level reference $p_0 = 20$ µPa is unimportant because of cancellation in the numerator and denominator of Equation (6).

NOTE 2 It is recommended that the maximum alternate background noise correction be bounded to invite use. A minimum value cap equal to the standard background noise correction is recommended for compatibility with past applications. A maximum value of 10 dB is recommended to prevent seemingly unrealistic corrections.

NOTE 3 A spreadsheet implementing the alternate background noise correction described in this Technical Report is available at: http://www.ecma-international.org/publications/techreports/E-TR-107.htm.

NOTE 4 The statistical background noise correction in [8] contains improvements relative to the correction K_S presented in this technical report: 1) the more accessible normal percentile is used instead of the VCS percentile of Equations (4) and (5) and 2) a minimum source-background noise difference ΔL_{min} is introduced to assure physical source descriptions in the presence of fluctuating background noise which may masquerade as a spurious source.

NOTE 5 The ability of the minimum source-background noise difference $\Delta L_{\min}$ [8] to protect against the situation of steady, high level background noise needs to be explored in future work. This situation may arise, for example, when using low-cost microphones and preamplifiers, whose instrumentation noise tends to be steady and high level and may be comparable to or higher than the level of the noise source under measurement.

Annex A
(normative)

Derivation of the Statistical Background Noise Correction

The derivation below has no guard band, while remaining mathematically equivalent.

A.1 Problem Statement

A source measurement m^2 has simultaneous contributions of the true source s^2 and background noise p_1^2:

$$m^2 = s^2 + p_1^2 \qquad \text{A.1}$$

Here the subscript "1" denotes the background noise sample included in the source measurement. The true source s^2 and background noise sample p_1^2 may fluctuate over time; any source and background fluctuations are statistically independent because the source and background phenomena are different phenomena and are unrelated to one another. Each of the terms in Equation (9) is a mean square, namely an average of the square of sound pressure over time. The true source may be estimated by removing a background noise estimate obtained from a separate measurement, or sample, of the background noise obtained when the source is not operating:

$$\begin{aligned} \hat{s}^2 &= m^2 - p_2^2 \\ &= s^2 + (p_1^2 - p_2^2) \end{aligned} \qquad \text{A.2}$$

in which $\hat{s}^2$ is an estimate of the source and p_2^2 is a separately measured background noise sample. Since background noise samples p_1^2 and p_2^2 are distinct and different, their difference may be positive or negative. Source estimate $\hat{s}^2$ may therefore either be above or below the true source s^2 and has an accuracy that depends on the statistical behaviour of the background noise sample difference $p_1^2 - p_2^2$, which is discussed in the next section.

A.2 Background noise distribution

A statistical distribution is derived from a background noise model well established in the literature. Background noise sound pressure p is represented as an aggregation of sinusoids of various frequencies:

$$p = \sum_{m=1}^{M} p_m \qquad \text{A.3}$$

Each sinusoid p_m is taken to be the resultant of multiple contributions of like frequency and random amplitude and phase

$$p_m = \sum_{n=1}^{N_m} A_{mn} \cos(\omega_m t + \phi_{mn}) \qquad \text{A.4}$$

Here m indexes frequency components and n indexes phased sinusoidal contributions at like frequency ω_m. The amplitudes A_{mn} and phases ϕ_{mn} are random and independent of one another, the latter being uniformly distributed over $(0,2\pi)$. The number M of frequencies and the number N_m of phase contributions at each frequency are assumed large in a statistical sense. The background noise is the mean square of background noise sound pressure p and is obtained by combining, squaring and time-averaging the expressions in Equation (11). Since each sound pressure component p_m has unique frequency, the mean square of Equation (11) has the simple form

$$p^2 = \sum_{m=1}^{M} p_m^2 \qquad \text{A.5}$$

Consider the average of a term p_m^2 in Equation (13) over a time scale longer than the period $2\pi/\omega_m$ but shorter than the time scale of ambient fluctuation. Squaring Equation (12) produces a double summation with terms containing squared terms $\cos^2(\omega_m t)$ and $\sin^2(\omega_m t)$ and cross terms $\sin(\omega_m t)\cos(\omega_m t)$. Averaging reduces the squared terms to one-half and zeros the cross terms, giving

$$\frac{2p_m^2}{N_m\sigma_\xi^2} = \left(\frac{\sum_n A_{mn}\cos\phi_{mn}}{\sqrt{N_m}\sigma_\xi}\right)^2 + \left(\frac{\sum_n A_{mn}\sin\phi_{mn}}{\sqrt{N_m}\sigma_\xi}\right)^2 \qquad \text{A.6}$$

where σ_ξ^2 is the variance of the random variables $\xi_{1n} = A_{mn}\cos\phi_{mn}$ and $\xi_{2n} = A_{mn}\sin\phi_{mn}$. The normalization allows identification of the parenthetical quantities $\sum_n \xi_{kn}/(N_m^{1/2}\sigma_\xi)$, $k = 1,2$ as normally distributed variables with zero mean and unit variance, by the Central Limit Theorem[2]. Moreover, the variable $2p_m^2/(N_m\sigma_\xi^2)$ follows a Chi-Square distribution with 2 degrees of freedom, by definition of the Chi Square distribution[2]. The variance $\sigma_\xi^2 = Var[\xi_{kn}] = Var[A_{mn}\cos\phi_{mn}] = E(A_{mn}^2\cos^2\phi_{mn}) - [E(A_{mn}\cos\phi_{mn})]^2$. Here $E(\cdot)$ is the expectation operator and $Var(\cdot)$ is the variance operator. Since the amplitude A_{mn} and phase ϕ_{mn} are independent, the variance is $E(A_{mn}^2)E(\cos^2\phi_{mn}) - [E(A_{mn})E(\cos\phi_{mn})]^2$. Defining $\overline{A_m^2} \equiv E(A_{mn}^2)$ and noting $E(\cos^2\phi_{mn}) = \frac{1}{2}$ and $E(\cos\phi_{mn}) = 0$ leads to $\sigma_\xi^2 = \frac{1}{2}\overline{A_m^2}$. The foregoing yields

$$y_m \equiv \frac{4p_m^2}{N_m\overline{A_m^2}} \sim \chi_2^2 \qquad \text{A.7}$$

in which the symbol ~ means “goes as” or “follows”. In other words, the average of the square of a sinusoid made of multiple contributions of like frequency and independent random amplitude and phase follows a Chi-Square distribution χ_2^2 with two degrees of freedom, which happens to be an exponential distribution[2]. This result has been established for room and ocean acoustics[7,8].

Since background noise is typically broadband, not tonal, we now seek the distribution of the aggregation of the multiple sinusoidal components, each of unique frequency and comprised of multiple contributions with various phases, expressed in Equation (13). Also using Equation (15) reveals

$$p^2 = \sum_{m=1}^{M} p_m^2 = \frac{1}{4}\sum_{m=1}^{M} N_m\overline{A_m^2}y_m \qquad \text{A.8}$$

Each component p_m^2 has unique frequency and results from multiple randomly phased sinusoidal components. If the number N_m and mean square amplitude $\overline{A_m^2}$ vary such that the product $N_m\overline{A_m^2}$ varies slowly across the analysis frequency resolution, an approximation to the product may be removed from the summation giving

$$w \equiv \frac{4p^2}{N\overline{A^2}} \approx \sum_{m=1}^{M} y_m \sim \chi_{2M}^2 \qquad \text{A.9}$$

in which $N\overline{A^2}$ is the average of the product $N_m \overline{A_m^2}$ over frequency, and the closed property of the Chi-Square distribution has been used: $\chi_i^2 + \chi_j^2 = \chi_{(i+j)}^2$ [2]. The foregoing reveals that background noise p^2 follows approximately a Chi-Square distribution with $\nu = 2M$ degrees of freedom, where M is the number of frequency components in the background noise.

A.3 Background noise statistics

The background noise steadiness may be obtained by Equation (17) and the properties of a Chi-Square random variable. Since a degree ν Chi-Square random variable has mean ν and variance 2ν [2], the steadiness is given by

$$M = \frac{E(\chi_{2M}^2)^2}{Var(\chi_{2M}^2)} = \frac{\mu_{p^2}^2}{\sigma_{p^2}^2} \approx \frac{\hat{\mu}_{p^2}^2}{\hat{\sigma}_{p^2}^2} \qquad \text{A.10}$$

in which μ_{p^2} and $\hat{\mu}_{p^2}$ are the true and estimated mean of the background noise, and $\sigma_{p^2}^2$ and $\hat{\sigma}_{p^2}^2$ are the true and estimated variance of the background noise. The steadiness parameter is also the number of frequency components in the background noise by Equation (11). The steadiness may be determined experimentally from measured estimates of the mean and variance of background noise, as the right hand side of Equation (18) shows. Note that the statistics apply to the background noise, which is a mean square, not the background noise level.

A.4 Distribution of the background noise sample difference

Assuming background noise samples are drawn from the same background noise process, the background noise difference $p_1^2 - p_2^2$ involves two Chi-Square distributed variables with identical degrees of freedom. For statistically independent samples, the difference follows a special case of the Variance-Gamma distribution. This may be seen using the moment generating function (MGF) $M_W = E(e^{tW})$ [2]. The difference

$$u \equiv w_1 - w_2 = \frac{2M}{\mu_{p^2}}\left(p_1^2 - p_2^2\right) \qquad \text{A.11}$$

has a MGF of $M_u = E[e^{t(w_2 - w_1)}] = E(e^{tw_2})E(e^{-tw_1}) = M_{w_2}(t)M_{w_1}(-t)$ since w_1 and w_2 are independent. For a Chi-Square variable of degree ν, the MGF is $M_W = (1-2t)^{-\nu/2}$ [2] and

$$M_u(t) = (1-4t^2)^{-\nu/2} = \left(\frac{1/4}{1/4 - t^2}\right)^{\nu/2} \qquad \text{A.12}$$

This expression turns out to be a special case of the Variance-Gamma distribution[5], which has probability density function (PDF) $\phi_{VG}(x)$ and MGF $M_{VG}(t)$

$$\begin{aligned} \phi_{VG}(x) &= \frac{(\alpha^2 - \beta^2)^{\lambda}}{\sqrt{\pi}\,\Gamma(\lambda)(2\alpha)^{\lambda - \frac{1}{2}}} \, |x - \mu|^{\lambda - \frac{1}{2}} \, K_{\lambda - \frac{1}{2}}\left(\alpha\,|x - \mu|\right)\exp\left(\beta\,|x - \mu|\right) \\ M_{VG}(t) &= e^{\mu t}\left[\frac{\alpha^2 - \beta^2}{\alpha^2 - (\beta + t)^2}\right]^{\lambda} \end{aligned} \qquad \text{A.13}$$

For $\mu = \beta = 0$, $\alpha = 1/2$, and $\lambda = \nu/2$ the MGF reduces to Equation (20), the mean $\mu = 0$, and the PDF reduces to

$$\phi_{VCS}(u) = \frac{1}{2^{\nu}\sqrt{\pi}\Gamma(\nu/2)} |u|^{(\nu-1)/2} K_{(\nu-1)/2}\left(\tfrac{1}{2}|u|\right) \qquad \text{A.14}$$

in which ν is degrees of freedom. We call this distribution the Variance Chi-Square, since it is a special case of the Variance-Gamma distribution. Here Γ is the Gamma function and K_{λ} is a modified Bessel function of the second kind. A Variance Chi-Square random variable is symmetric about $u = 0$, indicating that a background noise sample difference is positive or negative with equal likelihood.

A.5 Approximate normal distribution of the background noise sample difference

For a large number of independent frequency components, the background noise sample difference follows a normal distribution. To see this, evaluate the difference variable u of Equation (19) using Equation (17):

$$u = w_2 - w_1 = \sum_{k=1}^{K} y_k - \sum_{l=1}^{L} y_l \qquad \text{A.15}$$

The expression may be rewritten as:

$$\frac{u}{\sqrt{M}\sigma_y} = \left(\frac{\sum_k y_k - \mu_y}{\sqrt{M}\sigma_y}\right) - \left(\frac{\sum_l y_l - \mu_l}{\sqrt{M}\sigma_l}\right) \qquad \text{A.16}$$

Each of the two parenthetical terms follows a normal distribution with zero mean and unit variance, by the Central Limit Theorem[2]. The standard deviation $\sigma_y = 2$ because y is Chi-Square with two degrees of freedom[2]. The difference of the two terms follows a normal distribution with zero mean and variance two, by the summation property of the normal distribution[2]. The normalized variable $z \equiv u/\left(\sqrt{2M}\sigma_y\right) = u/\sqrt{8M}$ therefore follows a zero mean, unit variance Normal distribution[2]. The normal approximate PDF for the background noise sample difference is therefore

$$\phi(u) \approx \frac{1}{\sqrt{8M}} \phi_N\left(\frac{u}{\sqrt{8M}}\right) \qquad \text{A.17}$$

in which $\phi_N(z)$ is the standard normal distribution. Figure A.1 plots the exact VCS distribution, Equation (21), and its normal approximation and shows the approximation to be quite accurate for $M \geq 10$ frequency components. Figure A.2 compares the 95th percentile of the VCS and approximate normal distributions. The approximate normal percentile is within 1% of the VCS percentile.

A.6 Statistical background noise correction

To form the statistical background noise correction, Equations (10) and (19) are arranged to express the source estimate

$$\hat{s}^2 = s^2 + \frac{u_\alpha}{2M}\mu_{p^2} \qquad \text{A.18}$$

in which the percentile u_α at confidence value α is associated with the background noise sample difference distribution by Equation (5). Noting $s^2 = m^2 - p_1^2$ by Equation (10), normalizing by $\hat{s}^2$, and approximating the background noise sample p_1^2 and the background noise mean μ_{p^2} by the background noise mean estimate $\hat{\mu}_{p^2}$ gives

$$\frac{\hat{s}^2}{m^2} = 1 - \frac{\hat{\mu}_{p^2}}{m^2}\left(1 - \frac{u_\alpha}{2M}\right) \quad \text{A.19}$$

In terms of levels, the relationship is

$$\overline{L_p} = \overline{L'_{p(\mathrm{ST})}} + 10\lg\left[1 - 10^{-\Delta/10}\left(1 - \frac{u_\alpha}{2M}\right)\right] \quad \text{A.20}$$

in which $\overline{L_p}$ is background noise-corrected source level (BNCSL), $\overline{L'_{p(\mathrm{ST})}}$ is measured source level, and $u_\alpha = \Phi^{-1}(\alpha)$, where Φ^{-1} is the inverse cumulative probability distribution. Comparing Equations (1) and (28) reveals the statistical background noise correction, Equation (4).

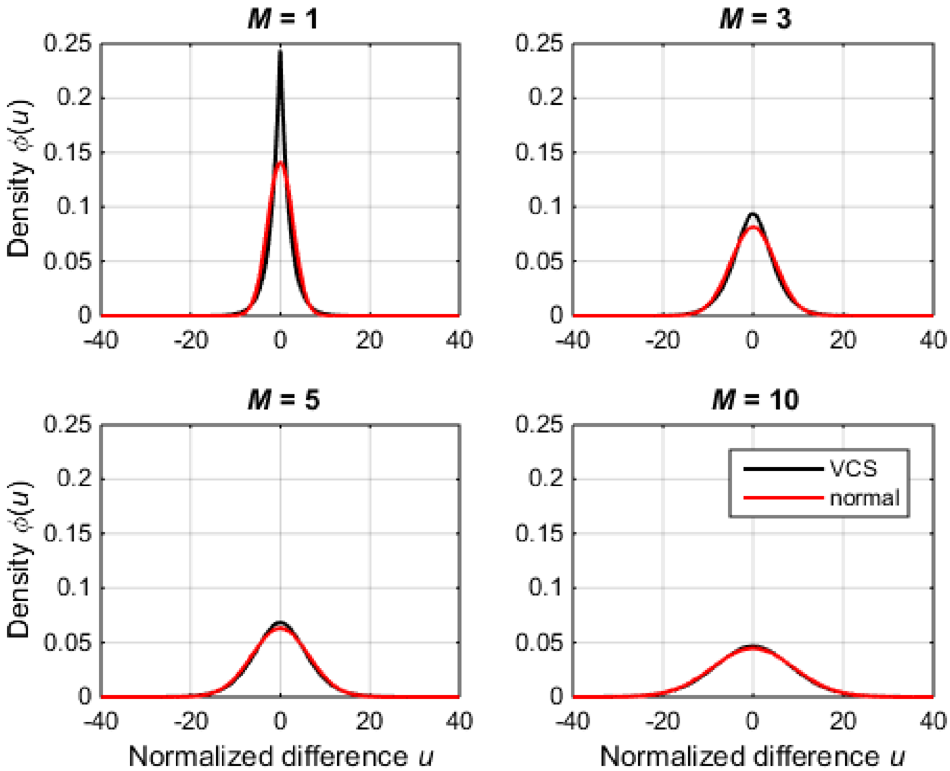


Figure A.1 — VCS and approximate normal probability density distributions

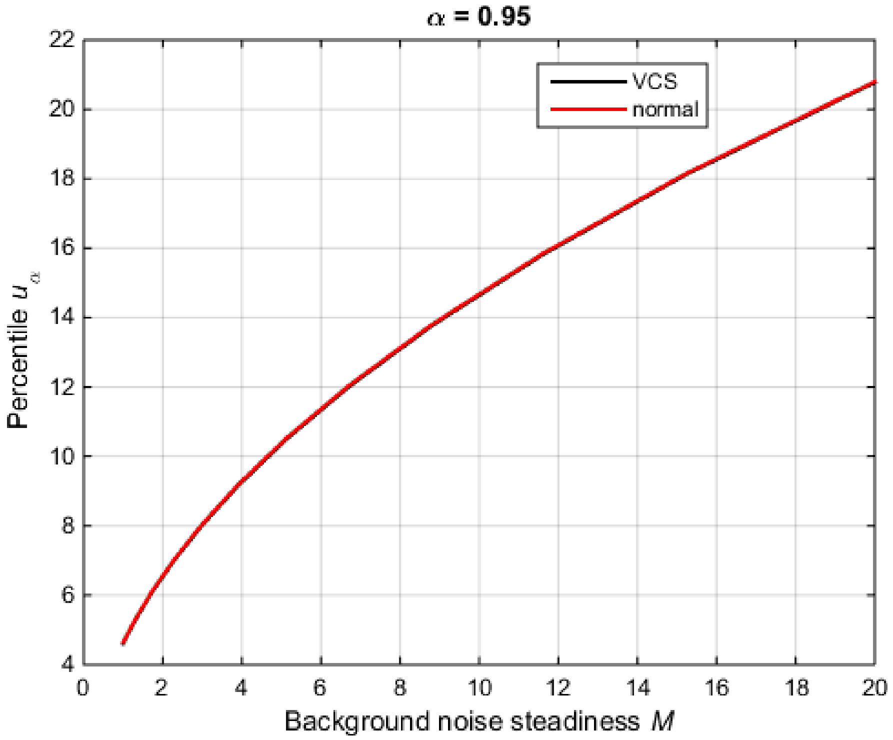


Figure A.2 — VCS and approximate normal percentiles, 95% confidence

Annex B
(informative)

Case Studies and Examples

Alternate and standard background noise corrections are compared for background noise sampled in actual manufacturer and hypothetical example chambers. The identity of the manufacturer chambers is not revealed at the request of certain manufacturers. Successively measured background noise samples were selected to represent background noise sampling that might occur around source measurements. The background noise data should not be construed to represent future background noise conditions, because background noise may change over time. Background noise data for two example chambers, A and B, were synthesized to contrast with the lower and more repeatable background noise data of the actual anechoic chambers.

Background noise statistics are given in Table B.1. Background noise steadiness M increases with background noise repeatability. Manufacturer A has the lowest and least steady background noise of the three chambers; however, some of the variation originates from rounding of background noise level to the nearest 0,1 dB. Manufacturer C has the highest and most steady background noise.

Table B.1 also shows two sets of background noise samples for hypothetical chambers, labelled “Example A” and “Example B”. The M values for these hypothetical chambers are more than an order of magnitude smaller than those of the manufacturer chambers.

Background noise corrections K_1 and K_{1A} for the 95% confidence value are plotted for the actual and example chambers in Figure B.1. The values of the alternate correction have not been limited by the application of limits in Equation (3). The alternate correction K_{1A} depends not only on the measured source-background noise level difference Δ, like the standard correction K_1, but also on background noise steadiness M; see Equations (6)-(8). Curves for K_{1A} are shown for the Manufacturer A and Manufacturer C. The K_{1A} values for Manufacturer B are omitted because they are nearly identical to those of Manufacturer C. The figure also shows the K_1 correction with caps $K_{1\max}$ of 0,46 dB and 1,26 dB, as prescribed by various standards [1,2,3,4].

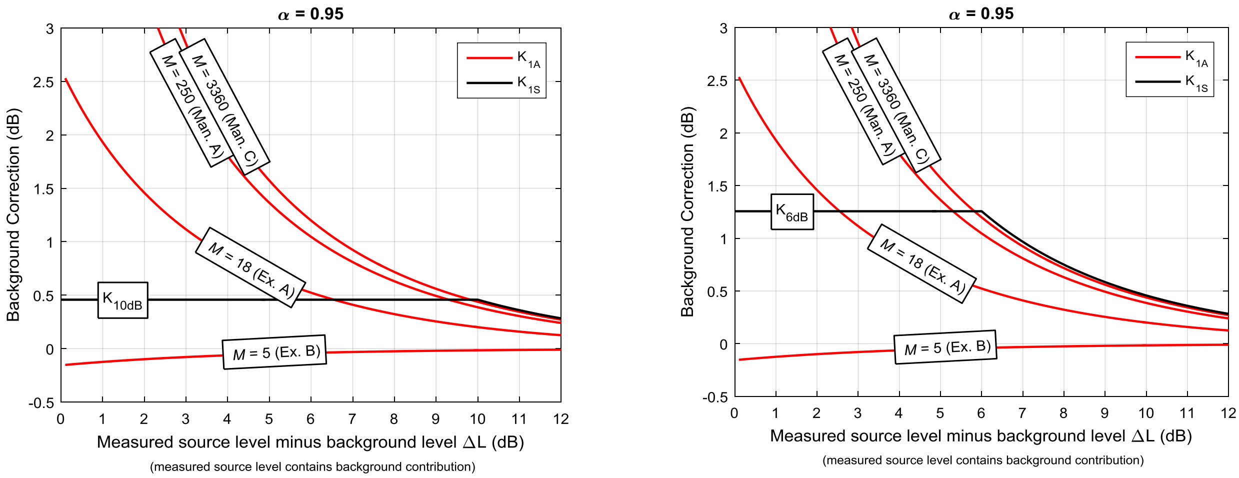
The behaviour of K_{1A} relative to K_1 highlights the assumptions underlying each background noise correction. As background noise steadiness increases, the condition of zero background noise variation is approached. In Figure B.1, K_{1A} approaches K_1 as and background noise steadiness M increases towards the condition of completely steady background noise with zero variation that is tacitly built into the standard correction K_1.

In reality background noise is unsteady, varying over time, and measured background noise may differ from the background noise contribution in a measured source level. Through its description of background noise variation, the background noise correction K_{1A} provides several benefits over the standard correction K_1:

- K_{1A} minimizes risk of understating true source level by providing a background noise-corrected source level (BNCSL) that upper bounds the true source level with 95% confidence, providing more assurance than the 50% confidence of the standard correction, when it is uncapped.
- K_{1A} provides a single background noise correction across all chamber grades, unlike the chamber grade dependent standard background noise correction [1,2,3,4], which results in a chamber grade penalty.
- K_{1A} produces lower BNCSL values than K_1 for steady background noise within 6 dB of measured source level for engineering grade and within 10 dB of measured source level for precision Grade 1. For Grade 1, BNCSL is approximately 2 dB lower when the measured source level is 3 dB above background level because of equal source and ambient contributions; for engineering Grade 2 they are about 1 dB lower. For source contributions below a steady background, the alternate background correction provides background corrected source levels more than 1 to 2 dB lower, depending on the grade, and as much as 10 dB lower than those provided by the standard background correction K_1.

Table B.1 — Background noise steadiness by chamber

Chamber	Background Noise Level (dB)				Background Noise (20 μPa)[2]		Background Noise Steadiness M
	Sample 1	Sample 2	Sample 3	Sample 4	Mean	Standard Deviation	
Manufacturer A	6,9	7,0	7,5	7,3	5,2	0,33	250
Manufacturer B	16,56	16,75	16,56	16,57	45,8	0,975	2210
Manufacturer C	21,74	21,78	21,76	21,91	151	2,61	3360
Example A	30,0	30,8	31,6	32,4	1300	320	18
Example B	40,0	41,5	43,0	44,5	18000	7900	5,2



a) K_{1S} for precision Grade 1

b) K_{1S} for engineering Grade 2

Figure B.3 — Alternate K_{1A} and standard K_{1S} background noise corrections

Annex C
(informative)

Calculation of Background Noise Steadiness

The background noise steadiness M is calculated from samples of background noise. Background noise is the average over time of squared pressure and is expressed in squared Pascals. Background noise is different than background noise level, which is expressed in dB.

The steps for obtaining the steadiness M are:

- Determine the background noise mean $\hat{\mu}_{p^2}$ by Equation (7)
- Determine the background noise standard deviation $\hat{\sigma}^2_{p^2}$ by Equation (8)
- Determine the steadiness M by Equation (6)

These steps use background noise, not background noise level. Background noise is related to background noise level by $p_i^2 = p_0^2 10^{L_i/10}$ in which p_i^2 is background noise, L_i is background noise level, and p_0 is the level reference. Although the level reference is standardized as 20 μPa, the convenient value of unity may be used instead because the level reference cancels in the ratio of Equation (6).

An example of determining background noise steadiness is given in Table C.1, using the data from Table B.1.

Table C.1 — Background noise by chamber

Chamber	Background Noise Level (dB)				Background Noise [multiples of (20 μPa)^2]						Stead-iness
	S1	S2	S3	S4	S1	S2	S3	S4	Mean	Std. dev.	
Manufacturer A	6.9	7.0	7.5	7.3	4.9	5.0	5.6	5.4	5,2	0,33	250
Manufacturer B	16.56	16.75	16.56	16.57	45	47	45	45	45,8	0,975	2210
Manufacturer C	21.74	21.78	21.76	21.91	149	151	150	155	151	2,61	3360
Example A	30,0	30,8	31,6	32,4	1000	1202	1445	1738	1300	320	18
Example B	40,0	41,5	43,0	44,5	10000	14125	19953	28184	18000	7900	5,2

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